

Application No. 09/678,058  
Page 10

### REMARKS

Claims 1-13 were examined and reported in the Office Action. Claims 1-13 are rejected. Claim 3 is cancelled. Claims 1-2, and 4-13 are amended. Claims 1-2, and 4-13 remain.

Applicant requests reconsideration of the application in view of the following remarks.

#### I. Drawing Objections

It is asserted in the Office Action that the drawings are objected to under 37 CFR §1.83(a). Applicant submits new figures 1 and 2 to overcome the 37 CFR §1.83(a) objection. Applicant submits that the specification as filed describes the details of the matrix driving scheme in sufficient detail for a person skilled in the art to practice the invention without need of illustration by a drawing(s). However, in view of the assertion in the Office Action that a drawing correction is required in reply to the Office Action to avoid abandonment of the application, Applicant submits Figures 1 and 2. Figure 1 illustrates a typical waveform for a common driving signal. Figure 2 shows matrix A, representing an orthogonal block circular matrix. Approval is respectfully requested.

#### II. Claims Rejected Under 35 U.S.C. §112, first paragraph

It is asserted in the Office Action that claims 1-13 are rejected under 35 U.S.C. §112, first paragraph, as containing subject matter which was not described in the specification in such a way as to enable one skilled in the art to which it pertains. Applicant respectfully disagrees.

In response to the assertions in the Office Action, the claims have been amended and are now directed to a driving method for a passive matrix LCD. Claim 1 includes the limitations that the addressing function (row (common) driving matrix) is represented by an orthogonal block circulant matrix and Claim 3 is cancelled. The

A

Application No. 09/678,058

Page 11

support for the above-mentioned amendments can be found in cancelled claim 3 and on pages 1 and 4 of the specification, as originally filed.

Applicant submits that the description as filed is full, clear, concise and sufficient to enable a person skilled and familiar in the art of display technology to make and use the invention. Applicant refers to the documents; "Hadamard Matrix Analysis and Synthesis" by Rao Yarlagadda and Hershey, "Circulant Matrices" by Philip J Davies, "Paraunitary Matrix Driving Scheme for Liquid Crystal Display" by Yeung et al and "Nonlinear Programming" by Avriel, as examples of prior art documents (all published before the priority date of the present application) that would be well known to the skilled person, and for which Applicant considers form part of the skilled person's general knowledge. (Applicant includes the above-mentioned documents in an information disclosure document).

Additionally, regarding the rejection to claims 2 and 5, Applicant submits that row interchanges and column interchanges of an orthogonal matrix are from part of a well-known and well-practiced technique in the relevant industry. For example, the application sufficiently describes the details of the construction of a matrix of scan data for the skilled person to use the invention. Row and column interchanges used as modifications for practical implementation of the matrix of scan data is already described in U.S. Patent No. 5,861,869 issued to Scheffer et al. at column 7, paragraph 6, by *"..... row functions can be generated from the above mentioned ones by interchanging matrix rows, negating matrix rows (i.e. multiplying them by -1), interchanging matrix columns, negating matrix columns, or any possible combinations of all four of these operations."*

Moreover, the underlying mathematical explanation is given in prior publications and mentioned in the present application.

Application No. 09/678,058

Page 12

It is also asserted in the Office Action that the order four and order eight orthogonal block-circulant building blocks are insufficiently described in the application. The order four and order eight orthogonal block-circulant building blocks, however, are given generally in the form of

$$E = [A_1 \ A_2 \ \dots \ A_N]$$

$E = [\dots]$  on page 5 of the specification and form a square block-circulant matrix as known in the art. Moreover, two order-four examples are illustrated in the application at the top of pages 6 and 7 as originally filed expanding from  $E_1$  and  $E_2$  into  $B_1$  and  $B_2$  on page 7.

$$E_1 = \begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & -1 & -1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & -1 & -1 \end{bmatrix}, B_2 = \begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

The Applicant considers it routine for a person skilled in the art of LCD driving schemes to obtain other matrices and orders themselves from the examples of the circulant form in the block-circulant matrix definition, the orthogonal block circulant matrix  $B_2$  as the examples demonstrate clearly how to apply the idea to other paraunitary matrices once the first block row has been given.

Application No. 09/678,058

Page 13

Accordingly, withdrawal of the 35 U.S.C. §112, first paragraph rejection for claims 1-13 are respectfully requested.

**III. Claims Rejected Under 35 U.S.C. §112, second paragraph**

It is asserted in the Office Action that claim 3 is rejected under 35 U.S.C. §112, second paragraph as being indefinite for failing to particularly point out and distinctly claim the subject matter which Applicant regards as the invention. Applicant has canceled claim 3.

Accordingly, withdrawal of the 35 U.S.C. §112, second paragraph rejection for claim 3 is respectfully requested.

**IV. Claims Rejected Under 35 U.S.C. §102(e)**

It is asserted in the Office Action that claims 1, 3, 4 and 6 are rejected under 35 U.S.C. §102(e) as being anticipated by U. S. Patent No. 6,054,972 issued to Otani et al. ("Otani"). Applicant respectfully disagrees.

According to MPEP §2131, "[a] claim is anticipated only if each and every element as set forth in the claim is found, either expressly or inherently described, in a single prior art reference.' (Verdegaal Bros. v. Union Oil Co. of California, 814 F.2d 628, 631, 2 USPQ2d 1051, 1053 (Fed. Cir. 1987)). 'The identical invention must be shown in as complete detail as is contained in the ... claim.' (Richardson v. Suzuki Motor Co., 868 F.2d 1226, 1236, 9 USPQ2d 1913, 1920 (Fed. Cir. 1989)). The elements must be arranged as required by the claim, but this is not an ipsissimis verbis test, i.e., identity of terminology is not required. (In re Bond, 910 F.2d 831, 15 USPQ2d 1566 (Fed. Cir. 1990))."

Applicant's amended claim 1 contains the limitations of "[a] driving method for a passive liquid crystal display, comprising: (i) a plurality of orthogonal addressing functions; wherein (ii) said plurality of orthogonal addressing function is applied simultaneously to a plurality of rows of the said display matrix; (iii) said plurality of

Application No. 09/678,058

Page 14

orthogonal addressing functions comprising a row (common) driving matrix and wherein; (iv) said plurality of orthogonal addressing functions is represented by an orthogonal block-circulant matrix."

Otani discloses a multi-line addressing driving scheme for an LCD display (see, for example, Otani, column 1, line 32 that asserts selecting "a plurality of scanning lines simultaneously"). The row waveform of the liquid crystal display is an orthogonal matrix of scan data, M (see Otani, column 1, line 42) this row waveform forms a key part of Otani. The scan matrix M of Otani is a Kronecker product of a Hadamard matrix, which in a mathematical context means the matrix is a block diagonal matrix. This is different to the orthogonal matrix of scan data with a block-circulant structure, as now claimed as the driving method in Applicant's amended claim1.

Further, Otani describes the Hadamard matrix (and its minor modifications (e.g. Formula 8 is the Kronecker product of Formula 11 and Formula 12 of Otani) as a "circulant Hadamard matrix" (see, e.g., Otani, column 2, line 47). However, Applicant submits that this "circulant Hadamard matrix" is not the "orthogonal block-circulant matrix" as now claimed and that Applicant's claimed invention is fundamentally different from the disclosure of Otani. Applicant points to the very standard and well-known definitions of "Block," "Circulant," and "Hadamard" in the context of mathematics (e.g. refer to reference [Circulant] and to reference [Hadamard www] for the properties of Hadamard matrix.)

Moreover, an advantage of the orthogonal block-circulant matrix of Applicant's claimed invention comprising the scan data is that the design only needs to input and store the scan data of the first block row into the memory and it is then possible to generate the entire matrix of scan data in a circulant fashion (or technique or formula). In contrast, there is no such circulant pattern for the Hadamard matrix taught in Otani. Others differences between Otani and Applicant's driving scheme is that the voltage levels required for the block-circulant matrix of scan data claimed need not be +1 or -1, whereas the voltage levels required for a Hadamard matrix must be so.

Application No. 09/678,058  
Page 15

Therefore, since Otani does not disclose, teach or suggest all of Applicant's amended claim 1 limitations, Applicant respectfully asserts that a *prima facie* rejection under 35 U.S.C. §102(e) has not been adequately set forth relative to Otani. Thus, Applicant's amended claim 1 is not anticipated by Otani. Additionally, the claims that directly or indirectly depend from Applicant's amended claim 1, namely claims 4 and 6 (claim 3 being cancelled), are also not anticipated by Otani for the same above reason.

Accordingly, withdrawal of the 35 U.S.C. §102(e) rejection for claims 1, 3, 4 and 6 are respectfully requested.

**V. Claims Rejected Under 35 U.S.C. §103(a)**

A. It is asserted in the Office Action that claims 2 and 5 are rejected under 35 U.S.C. §103(a) as being unpatentable over Otani in view of Applicant's admitted prior art ('AAPA'). Applicant respectfully disagrees.

According to MPEP §2142 "[t]o establish a *prima facie* case of obviousness, three basic criteria must be met. First, there must be some suggestion or motivation, either in the references themselves or in the knowledge generally available to one of ordinary skill in the art, to modify the reference or to combine reference teachings. Second, there must be a reasonable expectation of success. Finally, the prior art reference (or references when combined) must teach or suggest all the claim limitations. The teaching or suggestion to make the claimed combination and the reasonable expectation of success must both be found in the prior art, and not based on applicant's disclosure." (In re Vaeck, 947 F.2d 488, 20 USPQ2d 1438 (Fed. Cir. 1991)). Further, according to MPEP §2143.03, "[t]o establish *prima facie* obviousness of a claimed invention, all the claim limitations must be taught or suggested by the prior art. (In re Royka, 490 F.2d 981, 180 USPQ 580 (CCPA 1974)." "All words in a claim must be considered in judging the patentability of that claim against the prior art." (In re Wilson, 424 F.2d 1382, 1385, 165 USPQ 494, 496 (CCPA 1970), emphasis added.)

Applicant's claims 2 and 5 depend from Applicant's amended claim 1. Applicant's amended claim 1 is discussed above in view of Otani.

Application No. 09/678,058  
Page 16

In response to assertion in the Office Action in paragraph 12, Applicant comments that row and column interchanging for an orthogonal block-circulant matrix was not considered prior to the Applicant's claimed invention. The inventors exercised considerable inventive skill in recognizing the potential for transferring those ideas to the field of orthogonal block-circulant matrices and scan data. The use of interchanging was known to improve contrast, but the Applicant adapted the previous teaching for use as a driving scheme technique. The adaptation required was not an obvious, nor a simple process.

Applicant's technique provides a method of finding a suitable matrix using a cost or 'penalty function'  $P(E)$  in non-linear programming technique. The criteria for finding a matrix include the properties of (i) entries being +1 or -1; (ii) orthogonality; (iii) block-circulant property. Applicant's claimed invention provides a new matrix-driving scheme of considerably reduced complexity over the prior art. Applicant's driving scheme advantageously provides significant memory saving improvements over the known prior art techniques, this is particularly significant for large matrix orders and cannot be achieved in a Hadamard matrix such as that disclosed in Otani.

Since neither Otani, AAPA, nor the combination of the two teach, disclose or suggest the limitations contained in Applicant's amended claim 1, as listed above, there would not be any motivation to arrive at Applicant's claimed invention. Thus, Applicant's amended claim a is not obvious over Otani in view of AAPA since a *prima facie* case of obviousness has not been met under MPEP §2142. Additionally, the claims that directly or indirectly depend from amended claim 1, namely claims 2 and 5, would also not be obvious over Otani in view of AAPA for the same reason.

Accordingly, withdrawal of the 35 U.S.C. §103(a) rejection for claims 2 and 5 is respectfully requested.

A

Application No. 09/678,058

Page 17

B. It is asserted in the Office Action that claims 8-10 are rejected under 35 U.S.C. §103(a) as being unpatentable over Otani in view of U. S. Patent No. 4,993,075 issued to Sekihara et al. ("Sekihara"). Applicant respectfully disagrees.

Applicant notes that Sekihara is from the field of image processing, which can be considered as a different technological area requiring different knowledge according to a person skilled in the art from that of liquid crystal displays. Although the "non-linear programming" technique has been shown by others (e.g. Sekihara and reference [Nonlinear]) to be a workable and efficient method for image processing. Sekihara uses a mature non-linear programming technique in the image reconstruction of an NMR image to approximate to the real image. This is a different use in a different field to the Applicant's use and claimed technique of finding out numerically a suitable orthogonal block-circulant matrix of scan data for the row driving waveform.

Applicant has developed non-linear programming techniques applicable to liquid crystal displays. The developments were non-trivial and are not merely obvious modifications of Sekihara, they required the skilled person to exercise inventive skill, whether skilled in LCDs or imaging. It is not trivial for a person skilled in the area of image reconstruction to consider the applicability of non-linear programming to use an orthogonal block-circulant matrix in liquid crystal displays and to construct such a penalty function  $P(E)$  to find out the required matrices by numerical means.

Since neither Otani, Sekihara, nor the combination of the two teach, disclose or suggest the limitations contained in Applicant's amended claim 1, as listed above, there would not be any motivation to arrive at Applicant's claimed invention. Thus, Applicant's amended claim 1 is not obvious over Otani in view of Sekihara since a *prima facie* case of obviousness has not been met under MPEP §2142. Additionally, the claims that directly or indirectly depend from amended claim 1, namely claims 8-10, would also not be obvious over Otani in view of Sekihara for the same reason.

A



Application No. 09/678,058

Page 18

Accordingly, withdrawal of the 35 U.S.C. §103(a) rejection for claims 8-10 is respectfully requested.

CONCLUSION

In view of the foregoing, it is believed that all claims now pending, namely 1-2, and 4-13, patentably define the subject invention over the prior art of record and are in condition for allowance and such action is earnestly solicited at the earliest possible date.

If necessary, the Commissioner is hereby authorized in this, concurrent and future replies, to charge payment or credit any overpayment to Deposit Account No. 02-2666 for any additional fees required under 37 C.F.R. §§ 1.16 or 1.17, particularly extension of time fees.

A

## AN EFFICIENT LIQUID CRYSTAL DISPLAY DRIVING SCHEME USING ORTHOGONAL BLOCK-CIRCULANT MATRIX

The invention relates to a protocol for driving a liquid crystal display, particularly to a driving scheme of liquid crystal display, and more particularly to a special arrangement of the entries of the driving matrix, which results in an efficient implementation of the scheme and a reduction in hardware complexity.

Passive matrix driving scheme is commonly adopted for driving a liquid crystal display. For those high-mux displays with liquid crystals of fast response, the problem of loss of contrast due to frame response is severe. To cope with this problem, active addressing was proposed in which an orthogonal matrix is used as the common driving signal. However, the method suffers from the problem of high computation and memory burden. Even worse, the difference in sequences of the rows of matrix results in different row signal frequencies. This may result in severe crosstalk problems. On the other hand, Multi-Line-Addressing (MLA) was proposed, which makes a compromise between frame response, sequence, and computation problems. The block-diagonal driving matrix is made up of lower order orthogonal matrices. To further suppress the frame response, column interchanges of the driving matrix were suggested in such a way that the selections are evenly distributed among the frame. The complexity of the scheme is proportional to square of the order of the building matrix. Increase of order of the scheme results in complexity increase in both time and spatial domains. The order increase asks for more logic hardware and voltage levels of the column signal.

According to the invention there is provided a protocol for driving a liquid crystal display, characterised in that a row (common) driving matrix consists of orthogonal block-circulant matrices.

**Liquid Crystal Driving Scheme Using Orthogonal Block-Circulant Matrix**  
 The following shows an order-8 Hadamard matrix

$$\begin{bmatrix} 1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 \end{bmatrix}$$

As mentioned in the foregoing, because of the computation burden and sequency problem of using active driving, MLA was proposed. To implement an 8-way drive by using 4-line MLA, two order-4 Hadamard matrices are used as the diagonal building blocks of the 8x8 driving matrix. The resulting common driving matrix is as follows:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

3

To minimize the sequence problem, another 4x4 orthogonal building block has been proposed. The resulting row (common) driving matrix is as follows:

$$\begin{bmatrix} -1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & -1 \end{bmatrix}$$

A general  $m$ -way display will have a  $m \times m$  block diagonal orthogonal driving matrix made up of  $m/4$  (assuming that  $m$  is an integer multiple of 4) 4x4 building blocks. The actual voltage applied is not necessary  $\pm 1$  but a constant multiple of the value (i.e.,  $\pm K$ ). To further suppress the frame response, it has been proposed that column interchanges of the row (common) driving matrix such that the selections are evenly distributed among the frame. Using the 8-way drive as example, the following row (common) driving matrix results:

$$\begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

4

In the invention, there is proposed a method of generating orthogonal block-circulant building blocks that result in reduced hardware complexity of the driving circuitry. First of all, an orthogonal block-circulant matrix is defined as follows:

*Definition: An  $NM \times NM$  block-circulant matrix  $B$  consisting of  $N$   $M \times M$  building blocks  $A_1, A_2, \dots, A_N$  is of the form*

$$B = \begin{bmatrix} A_1 & A_2 & \dots & A_N \\ A_N & A_1 & \dots & A_{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ A_2 & A_3 & \dots & A_1 \end{bmatrix}$$

*It is said to be an orthogonal block-circulant if  $R^T R = R R^T = (NM) I_{NM}$ .*

For example, the following  $4 \times 4$  matrix is orthogonal block-circulant

In this case,  $N$  can be 2 or 4, if  $N=2$ , then each  $A_i$  is  $2 \times 2$  matrix. If  $N=4$ , then each  $A_i$  is

$$\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

A scalar (1 or -1). The orthogonal block-circulant matrix can be used as the diagonal building block of a row (common) driving matrix. By proper column and row interchanges, the resulting driving matrix has a property that each row is a shifted version of preceding rows and can be implemented by using shift registers. The following shows the resulting 8-way drive using  $4 \times 4$  orthogonal block-circulant matrix after suitable row and column interchanges.

$$\begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & -1 \end{bmatrix}$$

## 5

For higher order  $B$ , the choice of the order of sub-block  $A_1$  is limited. Some  $M$  might result in non-existence of orthogonal block-circulant  $B$ . Let  $MN=6$ , then  $M$ , the order of sub-block, can be 1, 2 or 3. It can be shown that orthogonal block-circulant  $B$  can be achieved by  $M=2, 3$ , but not  $M=1$ . In general, given that  $MN$  is even it can be shown that orthogonal block-circulant  $B$  always exists provided that  $M \neq 1$ . In the following, two means of generating orthogonal block-circulant matrices are proposed.

The first method is based on theory of *paraunitary* matrix but it by no means generates all orthogonal block-circulant matrices. The second method is a means to identify orthogonal block-circulant matrices by nonlinear programming.

Theoretically, it can be used to generate all orthogonal block-circulant matrices.

#### Generation of Orthogonal Block-Circulant Matrix Using Paraunitary Matrix

Consider order  $M \times NM$  sub-matrix of  $B$  as follows:

$$E = [A_1 \ A_2^* \ \dots \ A_N]$$

Define  $n \times n$  shift matrix  $S_{nm}$  as follows

$$S_{nm} = \begin{bmatrix} 0 & I_{nm} \\ 0_{(n-m) \times (n-m)} & 0 \end{bmatrix}$$

A paraunitary matrix  $E$  of order  $M \times NM$  satisfies

- (i)  $E$  is orthogonal i.e.,

$$EE^T = I$$

- (ii)  $E$  is orthogonal to its column shift by multiples of  $M$ , i.e.,

$$ES_{NM, iM}E^T = 0$$

For  $i = 1, 2, \dots, N-1$ .

In general, paraunitary matrices can be represented in a cascade lattice form with rotational angles as parameters.

6

The following two are two example 2x4 paraunitary matrixes.

$$E_1 = \begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix}$$

We have the following property of paraunitary matrices:

*Property:*  $B$  generated by block-circulating paraunitary  $E$  is orthogonal.

*Proof:* Define  $n \times n$  recurrent shift matrix  $R_{n,m}$  as follows

An orthogonal block-circulant matrix  $B$  of order  $NM \times NM$  with  $M \times N M$  sub-matrix  $E$  satisfies

(i)  $E$  is orthogonal i.e.,

$$EE^T = I$$

(ii)  $E$  is orthogonal to its recurrent shift by multiples of  $M$ . i.e.,

$$ER_{NM, iM}E^T = 0$$

For  $i = 1, 2, \dots, N-1$

Provided that  $E$  is paraunitary, as

$$R_{n,m} = S_{n,m} + S_{n-m,n-m}^T$$

we have

$$ER_{(N+1)M, iM}E^T = E(S_{n,m} + S_{n-m,n-m}^T)E^T = ES_{n,m}E^T + ES_{n-m,n-m}^TE^T = 0$$

and that completes the proof. Notice that  $E$  is paraunitary is a sufficient but not necessary condition for  $B$  to be orthogonal block-circulant. Using  $E_1$  and  $E_2$  as building blocks, we obtain the following orthogonal block-circulant matrices.

7

$$B_1 = \begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & -1 & -1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & -1 & -1 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

Notice that  $B_2$  is orthogonal circulant as well as orthogonal block-circulant. As illustrated before, by using it as the building block of row (common) driving matrix with suitable row and column interchanges, each row is a delay-1 shifted version of preceding row. However,  $B_1$  is orthogonal block-circulant but it is not circulant. By suitable row and column interchanges of the resulting driving matrix, two sets of row (common) driving waveforms are obtained. Within a set, each row is a shifted version of the others.

The complexity of implementation is proportional to the order of the sub-blocks  $A_i$  (i.e.,  $M$ ). For  $MM=4$ , we observe that  $M$  can be 1 or 2. For higher order,  $M=1$  does not result in any circulant  $B$  always exists and can be generated by  $2 \times 2N$  paraunitary matrices. The driving matrix resulted from  $B_2$  with suitable column interchanges is shown below:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 & -1 & 0 & -1 \end{bmatrix}$$

Rows 1, 3, 5, 7 and 2, 4, 6, 8 form the two sets within which each row is a shifted version of the others.



### Generation of Orthogonal block-circulant Matrix by Nonlinear Programming

We might also generate orthogonal block-circulant matrix by nonlinear programming. We use the method of steepest descent to illustrate the idea. The method of steepest descent is widely used in the identification of complex and nonlinear systems. The update law in identifying sub-matrix  $E$  can be stated as follows:

$$E_{n+1} = E_n + \delta \frac{\partial P}{\partial E}$$

Where  $\delta$  is the step size,  $P$  is the cost or penalty function. We set  $P$  as follows:

$$P(E) = \sum_{i,j} (e_{ij}^2 - 1)^2 + \|\bar{E}E^T - I\|_F^2 + \sum_r \|\bar{E}R_{2M,2M}E^T\|_F^2$$

$e_{ij}$  are the entries of  $E$ .  $\|\cdot\|_F$  is the Frobenius norm of a matrix. The first summation in the function forces all the entries of  $E$  to be  $\pm 1$ . The second one forces  $E$  to be orthogonal, while the third summation ensures orthogonal block-circulant property of the resulting  $B$ .

### List of Order-4 and Order-8 Orthogonal Block-Circulant Matrices

The following is an exhaustion of all  $2 \times 4$  and  $2 \times 8$  sub-matrices  $E$  with entries  $\pm 1$  that result in orthogonal block-circulant building block.

#### Order-4

$$(1) \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix};$$

$$(2) \begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix};$$

$$(3) \begin{bmatrix} -1 & 1 & -1 & -1 \\ 1 & 1 & -1 & 1 \end{bmatrix};$$

$$(4) \begin{bmatrix} -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & 1 \end{bmatrix};$$

9

(5) all alternatives of (1)-(4) generated by

- (i) sign inversion (i.e.,  $-E$ );
- (ii) row interchange, i.e.

$$\begin{bmatrix} 01 \\ 10 \end{bmatrix} E$$

- (iii) circulant shift of  $E$ , i.e.

$$ER_{4,2}$$

and any combinations of (i)-(iii).

Order-8.

$$(1) \begin{bmatrix} 1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 \end{bmatrix}$$

$$(2) \begin{bmatrix} 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

$$(3) \begin{bmatrix} 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 \\ 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

$$(4) \begin{bmatrix} 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 \end{bmatrix}$$

$$(5) \begin{bmatrix} -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \end{bmatrix}$$

$$(6) \begin{bmatrix} -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \end{bmatrix}$$

$$(7) \begin{bmatrix} -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

D

(8)

$$\begin{bmatrix} -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \end{bmatrix}$$

(9)

$$\begin{bmatrix} -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 \end{bmatrix}$$

(10)

$$\begin{bmatrix} -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

(11)

$$\begin{bmatrix} -1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 \end{bmatrix}$$

(12)

$$\begin{bmatrix} 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \end{bmatrix}$$

(13)

$$\begin{bmatrix} 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 \end{bmatrix}$$

(14)

$$\begin{bmatrix} 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \end{bmatrix}$$

(15)

$$\begin{bmatrix} 1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 \end{bmatrix}$$

(16)

$$\begin{bmatrix} 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 \end{bmatrix}$$

(17)

$$\begin{bmatrix} 1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 \end{bmatrix}$$

(18)

$$\begin{bmatrix} 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 \\ -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 \end{bmatrix}$$

II

(19)

$$\begin{bmatrix} 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 \\ -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 \end{bmatrix}$$

(20)

$$\begin{bmatrix} 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 \end{bmatrix}$$

(21)

$$\begin{bmatrix} 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 \end{bmatrix}$$

(22)

$$\begin{bmatrix} -1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 \end{bmatrix}$$

(23)

$$\begin{bmatrix} -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 \end{bmatrix}$$

(24)

$$\begin{bmatrix} -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 \\ -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 \end{bmatrix}$$

(25)

$$\begin{bmatrix} 1 & -1 & 1 & 1 & -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 \end{bmatrix}$$

(26)

$$\begin{bmatrix} 1 & -1 & -1 & -1 & -1 & 1 & -1 & -1 \\ -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 \end{bmatrix}$$

(27)

$$\begin{bmatrix} 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

(28) all alternatives of (1)-(27) generated by

- (i) sign inversion (i.e.,  $-E$ );
- (ii) row interchange, i.e.

$$\begin{bmatrix} 01 \\ 10 \end{bmatrix} E_1.$$

- (iii) circulant shift of  $E$ , i.e.

$$ER_{22}$$

$i=1, 2, \text{ or } 3$  and any combinations of (i)-(iii).

Thus using the invention a special arrangement of the entries of driving matrix is proposed. By imposing orthogonal block-circulant property to the building blocks of the row (common) driving waveform, the row signals can be made to differ by time shifts only. Each row can now be implemented as a shifted version of preceding rows by using shift registers. The complexity of the matrix driving scheme is greatly reduced and is linearly proportional to the order of the orthogonal block-circulant building block.

20

**ABSTRACT OF THE DISCLOSURE**

The invention relates to a protocol for driving a liquid crystal display, in which a row (common) matrix is made up of orthogonal block-circulant matrices which can be generated by nonlinear programming or alternatively by paraunitary matrixing.

$$\begin{bmatrix} 1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 \end{bmatrix}$$

~~Figure 2~~

Figure 1

JHW/12/21/04